In the above expressions, γ' is the magnitude of the magnetomechanical ratio, μ_0 is the magnetic inductive capacity of free space, M is the saturation magnetization, $\lambda_1 = \lambda/M$ is a reduced damping constant, and H is the applied magnetostatic field. The problem is that of determining the values of x for which χ_{xy}' is maximum and one half of its maximum value and the values of x for which $|\chi_{xy}|$ is maximum and $2^{-1/2}$ of its maximum value. Although this is a rather involved analytical procedure using the above expressions, a graphical solution can be obtained quite easily. In the present case, this was accomplished by calculating the values of F and $(F^2+G^2)^{1/2}$ as a function of x with a computer for values of λ_1 of 0.125, 0.100, 0.080, and 0.070. The results were then plotted as a function of x and the aforementioned values of x were obtained from the curves, one of which is shown in Fig. 2 for $\lambda_1 = 0.080$. The following values were obtained for x in this manner:

$$x_0 = 1 - \lambda_1^2/2, \qquad F_{\max} = F_C @ x = x_0$$

$$\left.\begin{aligned} x_2 &= 1 - \lambda_1 - \lambda_1^2/2, \\ x_5 &= 1 + \lambda_1 - 3\lambda_1^2/5, \end{aligned}\right\} F_0F_{2,5}^{-1} = 2 @ x = x_2, x = x_5 \quad (5a)$$

$$x_1 = 1 - \lambda_1^2 \qquad (F^2 + G^2)_{\max} = A_1 @ x = x_1$$

$$x_3 = 1 - \lambda_1 - \lambda_1^2$$

$$x_4 = 1 + \lambda_1 - 3\lambda_1^2/2, \; A_1A_{3,4}^{-1} = 2 @ x = x_3, x = x_4 \quad (5b)$$

where the x_i's are defined in Fig. 2. The true line width in terms of x is given as

$$\Delta x = x_5 - x_2 = 2\lambda_1 - \lambda_1^2/10.$$

The measured line width in the coupler in terms of x is

$$\widetilde{\Delta x} = x_4 - x_3 = 2\lambda_1 - \tfrac{1}{2}\lambda_1^2. \quad (6)$$

Thus, if one measures in the coupler the line width $\widetilde{\Delta x}$, the value obtained is 2 per cent larger than the true line width Δx, if the reduced damping constant is 0.10. As the reduced damping constant decreases below 0.10, the measured line width approaches the true line width even more closely. The actual expression for the reduced damping constant can be obtained from (5b), (6), and (4b) as

$$\widetilde{\Delta H} H_1^{-1} = \tfrac{1}{2}(4\lambda_1 - \lambda_1^2)(1 - \lambda_1^2)^{-1}$$

where $\widetilde{\Delta H} = H_4 - H_3$. This expression reduces to the following for λ_1 small:

$$\lambda_1 = \tfrac{1}{2}\widetilde{\Delta H}/H_1. \quad (3a)$$

The expression for the g factor can be obtained from (5a), (5b), and (4b) as:

$$g = 2H_rH_1^{-1}(1 - \lambda_1^2)(1 - \tfrac{1}{2}\lambda_1^2)^{-1} \simeq 2H_rH_1^{-1}(1 - \tfrac{1}{2}\lambda_1^2).$$

Acknowledgment

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Applications of Directional Filters for Multiplexing Systems*

FRANKLIN S. COALE†

Summary—**The design of microwave multiplexing systems for frequency channelization of a broad-band microwave spectrum is complicated by problems such as off-resonance mismatch and mutual interaction between adjacent filters. By employing directional filters as basic building blocks, it is possible to construct multiplexing filters with a perfect input match since the input VSWR of a directional filter is theoretically unity both at resonance and off-resonance. Less insertion loss of a manifold may be obtained by the use of directional filters than with conventional band-pass filters. Curves giving the predicted response of a manifold containing n elements are presented for single-tuned and double-tuned directional filters. An asymmetrical response shape is obtained which has a midband insertion loss related to the separation of adjacent channels.**

An experimental model consisting of a five-channel multiplexer has been constructed utilizing double-tuned-circular-waveguide directional filters.

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Introduction

MICROWAVE filters are used extensively in separating or combining signals of different frequencies. It is desirable in some applications to divide the frequency band into many narrow channels which are spaced in such a way that complete coverage is obtained; *i.e.*, any signal within the frequency band will be within the pass band of one or more filters. Most multiplexers of this variety have consisted of a number of filters loosely coupled to a transmission line (hereafter referred to as a manifold). Loose coupling has been necessary because of the off-resonant mismatch associated with most band-pass filters. When a great many filters are connected on a manifold it is possible to obtain a large mismatch at some frequency within the bandwidth of the system due to the quasi-periodic nature of the manifold. Tighter coupling to the manifold produces correspondingly greater off-resonance mismatch in addition to mutual interaction between filters.

By employing filters which have a constant input impedance (a unity input VSWR) as basic building blocks, it is possible to construct multiplexing filters with a perfect input match. Filters of this type called "directional filters" have been the subject of a number of recent papers.[1–7] A directional filter is a four-port network having the response shown in Fig. 1. A typical band-pass response is obtained between arms 1 and 4; a band rejection response, between arms 1 and 2; arm 3 is isolated from the input. Ideally, the input VSWR is unity both at resonance and off-resonance. This paper will consider the application of directional filters to multiplexing systems where adjacent bands of frequencies are separated in the manner illustrated in Fig. 2.[8]

In particular the point at which adjacent bands overlap will be chosen to be the half-power points. The filters will be placed in the manifold in the order of increasing frequency.

Theory of Single-Tuned-Directional Filter Manifolds

The response of the nth filter of a multichannel filter system containing single-tuned-directional filters may be obtained by forming a product of the band-rejection response of the $n-1$ filters (between the input and the nth filter) and the single filter may be given in terms of a normalized wavelength Ω and a normalized quality factor Q where

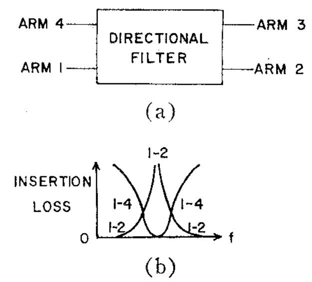

Fig. 1—Response of a directional filter.

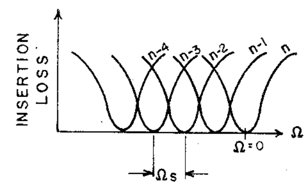

Fig. 2—Method of placing filters on a manifold.

$$\Omega = \frac{\lambda_{g0}}{\lambda_g} - 1$$

$$Q = Q_L\left(\frac{\lambda}{\lambda_g}\right)^2.$$

λ_{g0} is the resonant guide wavelength of the directional filter, λ_g and λ are the guide wavelength and free space wavelength of the input signal. Q_L is the loaded Q of the cavity. It will be assumed that the unloaded Q is infinite and that the cavity is loaded only by the coupling apertures.

If a signal of frequency corresponding to Ω and magnitude 1 is incident at arm 1 of a directional filter the following voltages exist at the other arms.[5]

$$E_1 = 1 \tag{1a}$$

$$E_2 = \frac{2\Omega Q}{(1 + 4\Omega^2 Q^2)^{1/2}} \tag{1b}$$

$$E_3 = 0 \tag{1c}$$

$$E_4 = \frac{1}{(1 + 4\Omega^2 Q^2)^{1/2}}\,. \tag{1d}$$

If one assumes that the Q's of adjacent filters do not vary appreciably from one another and also that there is a fixed separation of the resonant frequencies by an amount corresponding to Ω_s as shown in Fig. 2 where

$$\Omega_s = \frac{\lambda_{gK}}{\lambda_{gK-1}} - 1,$$

then the output from arm 4 of the nth filter is given by the product

$$E_n = E_{4_n} E_{2_{n-1}} E_{2_{n-2}} \cdots E_{2_1}.$$

Writing this in terms of Q, Ω and Ω_s this expression becomes

$$E_n = \frac{1}{(1 + 4\Omega^2 Q^2)^{1/2}} \prod_{\substack{a=1 \\ n\neq 1}}^{n-1} \frac{2[\Omega + a\Omega_s]Q}{[1 + 4(\Omega + a\Omega_s)^2 Q^2]^{1/2}}\,. \tag{2}$$

[1] F. S. Coale, "A traveling-wave directional filter," IRE Trans. on Microwave Theory and Techniques, vol. MTT-4, pp. 256–260; October, 1956.

[2] S. B. Cohn and F. S. Coale, "Directional channel-separation filters," Proc. IRE, vol. 44, pp. 1018–1024; August, 1956.

[3] S. B. Cohn and F. S. Coale, "Research on Design Criteria for Microwave Filters," Stanford Res. Inst., Menlo Park, Calif., Third Quart. Prog. Rep. under Signal Corps Contract No. DA 36-039 SC 64625, Project 1331; September 15–December 15, 1955.

[4] E. M. T. Jones, F. S. Coale, O. Heinz, J. K. Shimizu, and S. B. Cohn, "Research on Design Criteria for Microwave Filters," Stanford Res. Inst., Menlo Park, Calif., Fourth Quart. Prog. Rep. under Signal Corps Contract No. DA 36-039 SC-64625, Project 1331; December 15–March 15, 1956.

[5] S. B. Cohn, E. M. T. Jones, and F. S. Coale, "Research on Design Criteria for Microwave Filters," Stanford Res. Inst., Menlo Park, Calif., Sixth Quart. Prog. Rep. under Signal Corps Contract No. DA 36-039 SC-64625, Project 1331; July 15–November 15, 1956.

[6] C. E. Nelson, "Ferrite-tunable microwave cavities and the introduction of a new reflectionless, tunable microwave filter," Proc. IRE, vol. 44, pp. 1449–1455; October, 1956.

[7] C. E. Nelson, "Circularly polarized microwave cavity filters," IRE Trans. on Microwave Theory and Techniques, vol. MTT-5, pp. 136–147; April, 1957.

[8] The problem of cascaded inputs and outputs has been previously treated by Cohn, Jones, and Coale, *op. cit.*

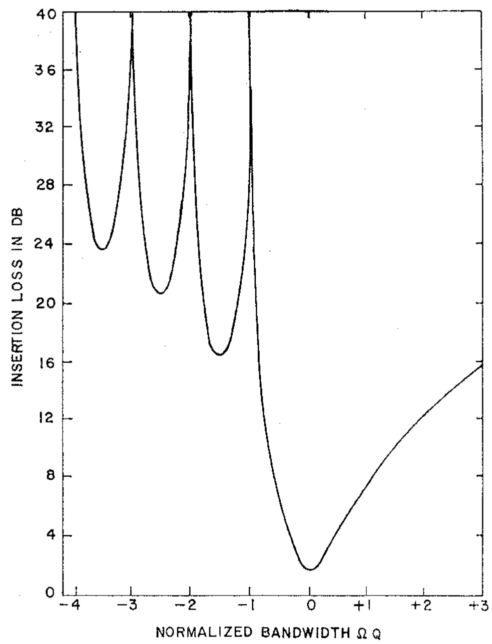

Fig. 3—Response of a single tuned directional filter manifold.

In general this expression may not be simplified and one must resort to numerical computation. If the value of Ω_s is given such that the responses of the filters intersect at their half-power points a curve shown in Fig. 3 results where insertion loss is plotted against ΩQ. At the resonant frequency of a single directional filter a zero exists in the output response as measured between arms 1 and 2. Hence zeros will be expected in the response of E_n at the resonant frequencies of the $(n-1)$ filters. At frequencies greater than the resonant frequency of the nth filter no zeros will exist (except at ∞) and a smooth monotonically decreasing curve is obtained which approaches the insertion loss curve of a single directional filter. The insertion loss at resonance may be deduced from (2) by setting $\Omega=0$

$$E_n\Big|_{\Omega=0} = \prod_{a=1}^{n-1} \frac{2a\Omega_s Q}{[1+4a^2\Omega_s{}^2Q^2]^{1/2}}.$$

If one allows n to approach ∞ and employs the relation

$$\frac{\sin \pi z}{\pi z} = \prod_{a=1}^{\infty}\left(1-\frac{z^2}{a^2}\right),$$

the following expression is found

$$\text{I.L.} = -10 \log \left[\frac{\pi}{2\Omega_s Q \sinh\left(\dfrac{\pi}{2\Omega_s Q}\right)}\right]. \tag{3}$$

Such a curve is shown in Fig. 4 as a function of insertion loss in db vs the normalized bandwidth, $\Omega_s Q$. For a value of $\Omega_s Q=1$ (corresponding to filters whose responses overlap adjacent filter responses at the 3-db points), an insertion loss of at least 1.66 db is expected.

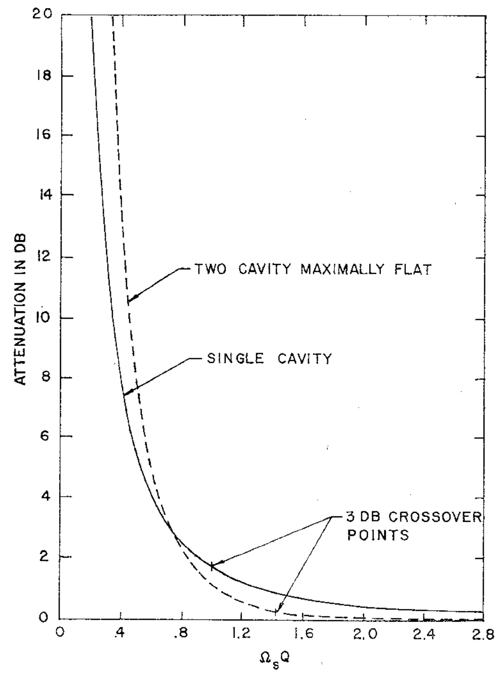

Fig. 4—Insertion loss of single and double tuned directional filter manifolds as a function of the Q-band separation constant Δ, Q.

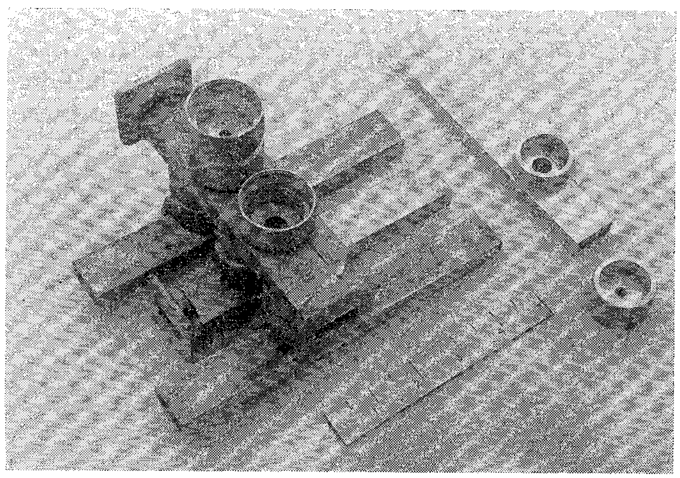

Fig. 5—Photograph of X-band experimental multiplexer.

Theory of Double-Tuned-Directional Filter Manifolds

The use of double-tuned-directional filters creates considerable sharper skirts on the band-pass response. The output response of the double-tuned-directional filter is given by[9]

$$E_4 = \frac{2K}{[(K^2+1)^2 + 8\Omega^2 Q^2(1-K^2) + 16\Omega^4 Q^4]^{1/2}} \tag{4}$$

(assuming $\Omega \ll 1$) where K is the coefficient of coupling normalized to the critical coefficient of coupling, and Q is the loaded Q of a single cavity; the Q's of the two

[9] This result is similar to that derived by Cohn, Jones, and Coale, *op. cit.*, and may be deduced from equations given in F. E. Terman, "Radio Engineers' Handbook," McGraw-Hill Book Co., Inc., New York, N. Y., sec. 3, para. 5, pp. 154–163; 1943.

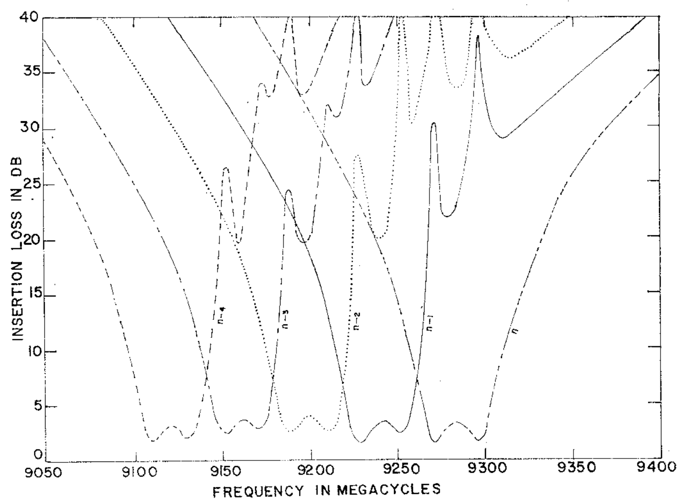

Fig. 6—Insertion loss of experimental multiplexer as a function of frequency.

cavities are assumed equal. E_2 is the complementary response of E_4 and given by

$$E_2 = \frac{K^2 - 1 - 4\Omega^2 Q^2}{[(K^2+1)^2 + 8\Omega^2 Q^2(1-K^2) + 16\Omega^4 Q^4]^{1/2}}. \quad (5)$$

The output response of the nth filter of the manifold is given by

$$E_n = \frac{2K}{[(K^2+1)^2 + 8\Omega^2 Q^2(1-K^2) + 16\Omega^4 Q^4]^{1/2}} \prod_{\substack{a=1 \\ n\neq 1}}^{n-1} \frac{-4Q^2(\Omega + a\Omega_s)^2 + K^2 - 1}{[(K^2+1)^2 + 8(\Omega + a\Omega_s)^2 Q^2(1-K^2) + 16Q^4(\Omega + a\Omega_s)^4]^{1/2}}. \quad (6)$$

Similar curves may be obtained as in the single tuned filters but double zeros exist where single zeros existed before. For the maximally flat case ($K=1$) the output voltage becomes:

$$E_n = \frac{1}{[1 + 4\Omega^4 Q^4]^{1/2}} \prod_{\substack{a=1 \\ n\neq 1}}^{n-1} \frac{-2Q^2(\Omega + a\Omega_s)^2}{[1 + 4(\Omega + a\Omega_s)^4 Q^4]^{1/2}} \quad (7)$$

and E_n at $\Omega = 0$ yields the insertion loss formula.

$$\text{I.L.} = -10 \log \left[\frac{\pi^2/(2Q^2\Omega_s^2)}{\sinh^2 \frac{\pi}{2Q\Omega_s} + \sin^2 \frac{\pi}{2Q\Omega_s}} \right]. \quad (8)$$

This is plotted in Fig. 4. Due to the sharper skirts of the double-tuned filter one should expect much less insertion loss than in the case of the single-tuned filter. The 3-db crossover point occurs at $Q\Omega_s = \sqrt{2}$ which corresponds to an insertion loss of 0.26 db.

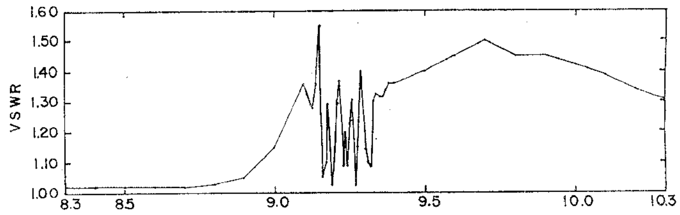

Fig. 7—VSWR of experimental multiplexer as a function of frequency.

Experimental Results

A five-channel multiplexer was constructed in the X-band region and is shown in Fig. 5. The particular design of the circular waveguide directional filters is given in the literature.[2–4] The bandwidth of each output response was approximately 40 mc. The curves are shown for the cavities in the reverse order, *i.e.*, the highest frequency cavity was placed first in the line. A curve showing the electrical performance is given in Figs. 6 and 7. It may be shown that if an elliptically polarized wave rather than a circular wave exists in the cavity that the input VSWR is not unity, nor is there a perfect band rejection response. Due to the particular manner of construction it is impossible to cause perfect circular polarization since the positions of coupling holes between the waveguide manifold and the circular cavities were not varied from filter to filter. Other causes for the rather high VSWR are due to inaccurate tuning of the cavities. The criterion chosen was response shape rather than input VSWR.

Conclusion

The use of directional filters in the design of complete-coverage multiplexers realizes maximum electrical performance. The theory presented in this paper has been substantiated by experiment.